Summer term 2019 Research Institute for Discrete Mathematics
Dr. U. Brenner University of Bonn

Linear and Integer Optimization
Assignment Sheet 11

1. Give an example each of

   (a) a full-dimensional unbounded rational polyhedron P such that P_I is empty.

   (b) an unbounded polyhedron P such that P_I is non-empty and bounded.

   (c) a polyhedron P such that $P_I \neq \emptyset$ is not closed.

   (d) a feasible and bounded integer linear program with no optimum solution. (2+2+2+2 points)

2. (a) Prove that a polyhedral cone is rational if and only if it is generated by a finite number of integral vectors. Conclude that $C_I = C$ for any rational cone C.

   (b) Let $P, Q \subseteq \mathbb{R}^n$ be two polyhedra. Show that $P_I + Q_I \subseteq (P+Q)_I$. Give an example where $P_I + Q_I \neq (P+Q)_I$ (3+2 points)

3. Let $P = \{x \in \mathbb{R}^{k+l} : Ax \leq b\}$ be a rational polyhedron (i.e. $A \in \mathbb{Q}^{m\times(k+l)}$, $b \in \mathbb{Q}^m$). Show that $\mathrm{conv}(P \cap (\mathbb{Z}^k \times \mathbb{R}^l))$ is a rational polyhedron. (4 points)

4. Show that each matrix $A \in \mathbb{Z}^{n\times n}$ with $\det(A) \in \{-1, 1\}$ arises from the identity matrix by a series of elementary unimodular column operations. (3 points)

Due date: Thursday, June 27, 2019, before the lecture.