Summer term 2019
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Linear and Integer Optimization
Assignment Sheet 8

1. Let $A \in \mathbb{Q}^{n\times n}$ be a regular matrix. Show that $\text{size}(A^{-1}) \leq 4n^2\text{size}(A)$. (2 points)

2. Define $\|A\| := \max_{\|x\|=1} \|Ax\|$ for $A \in \mathbb{R}^{n\times n}$, where $\|\cdot\| : \mathbb{R}^n \to \mathbb{R}$ is the standard Euclidean norm. Prove:
   (a) $\|A\|$ is a norm
   (b) $\|aa^T\| = a^T a$
   (c) $\|A\| = \max\{x^T Ax \mid \|x\| = 1\}$ if A is positive semidefinite
   (d) $\|A\| \leq \|A + B\|$ if A and B are positiv semidefinite.

   (2+2+2+2 points)

3. Let $P \subset \mathbb{R}^d$ be a finite set of points and let B be a ball containing P. Show: B is a minimum radius ball containing P if and only if the center of B lies in $\text{conv}(P \cap \partial B)$, where ∂B is the border of the ball. (5 points)

Due date: **Tuesday**, May 28, 2019, before the lecture.