

Linear and Integer Optimization

Assignment Sheet 5

1. Let $P = \{x \in \mathbb{R}^n \mid Ax \leq b\}$ be a polyhedron. Moreover, let

$$P^* := \{y \in \mathbb{R}^n \mid y^t x \leq 1 \text{ for all } x \in P\}$$

and

$$P^0 := \{y \in \mathbb{R}^n \mid y^t x \leq 0 \text{ for all } x \in P\}.$$

- (a) Show that P^* is a polyhedron.
(b) Prove that $(P^*)^* = P$ if and only if $b \geq 0$.
(c) In addition, assume $b = 0$. Show that $P^* = P^0$ and prove that P^0 is the convex cone generated by the rows of A . (2+3+2 points)
2. Let $P, Q \subseteq \mathbb{R}^n$ be polyhedra. Show that $P + Q$ is a polyhedron. (3 points)
3. Two sets $X, Y \subseteq \mathbb{R}^n$ are called *strictly separable*, if there is a hyperplane $\{x \in \mathbb{R}^n \mid a^t x = b\}$ such that $a^t x < b$ for all $x \in X$ and $a^t y > b$ for all $y \in Y$. Prove or disprove the following statement: Any two disjoint closed convex sets are strictly separable. (2 points)
4. Use the SIMPLEX ALGORITHM to solve the following linear programs:

(a)

$$\begin{array}{ll} \max & 2x_2 \\ \text{s.t.} & x_1 - x_2 \leq 4 \\ & -x_1 + x_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{array}$$

(b)

$$\begin{array}{ll} \max & 5x_1 + 3x_2 \\ \text{s.t.} & 4x_1 + 2x_2 \leq 12 \\ & 4x_1 + x_2 \leq 10 \\ & x_1 + x_2 \leq 4 \\ & x_1, x_2 \geq 0 \end{array}$$

(c)

$$\begin{array}{ll} \min & 5x_1 - x_2 \\ \text{s.t.} & x_1 - 3x_2 \leq 1 \\ & x_1 - 4x_2 \leq 3 \\ & x_1, x_2 \geq 0 \end{array}$$

(d)

$$\begin{array}{ll}\min & -x_1 - 2x_2 \\ \text{s.t.} & 2x_1 + x_2 \leq 5 \\ & -x_1 - x_2 \geq -3 \\ & x_1, x_2 \geq 0\end{array}$$

Show all intermediate simplex tableaus and describe why you can choose a specific variable to enter or leave the basis. If there is an optimum solution, also give its value. (2+2+2+2 points)

Due date: Thursday, May 9, 2019, before the lecture.

Ein Hinweis der Fachschaft Mathematik:

Die Fachschaft Mathematik feiert am 09.05. ihre Matheparty in der N8schicht. Der VVK findet am Mo. 06.05., Di. 07.05. und Mi. 08.05. in der Mensa Poppelsdorf statt. Alle weitere Infos auch auf fsmath.uni-bonn.de

English version:

The student council of mathematics will organize the math party on 9/05 in N8schicht. The presale will be held on Mon 6/05, Tue 7/05 and Wed 8/05 in the mensa Poppelsdorf. Further information can be found at fsmath.uni-bonn.de