Summer term 2019
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Linear and Integer Optimization
Assignment Sheet 4

1. In the job assignment problem, n jobs with execution times $t_1, ..., t_n \in \mathbb{R}_{\geq 0}$ need to be processed by m workers. For each job i we are given by $S_i \subseteq \{1, ..., m\}$ the set of workers that are qualified to perform job i. It is possible for several workers to process the same job in parallel to speed up the process but one worker can only process one job at a time.
   (a) Formulate an LP minimizing the *makespan* for processing all jobs (the time until the last worker finishes).
   (b) Dualize this LP. (2+2 points)

2. Prove that a polyhedron $P \subseteq \mathbb{R}^n$ is of dimension n if and only if P contains a vector x in its interior (i.e. there is some $\varepsilon > 0$ such that an n-dimensional ball $B = \{y \in \mathbb{R}^n \mid |y - x|_2 \leq \varepsilon\}$ with radius ε and center x is contained in P). (4 points)

3. Let $C_n := [-1, +1]^n$ be an n-dimensional hypercube. Determine the number f_k of k-dimensional faces for $k = 0 \ldots n$ and the total number of faces. Prove the correctness of your answers. (3 points)

4. For a polytope $P = \{x \in \mathbb{R}^n \mid Ax \leq b\} \neq \emptyset$ let $P' := \{(x, t) \in \mathbb{R}^n \times \mathbb{R} \mid Ax \leq tb, 0 \leq t \leq 1\}$.
   (a) Show that $P' = \mathrm{conv}((P \times \{1\}) \cup \{0\})$.
   (b) Prove that for each face F of P the set $\mathrm{conv}((F \times \{1\}) \cup \{0\})$ is a face of P'.
   (c) Do these these statements still necessarily hold if P is an unbounded polyhedron? (2+2+1 points)

5. Prove that any set $X \subseteq \mathbb{R}^n$ with $|X| > n + 1$ can be decomposed into subsets X_1 and X_2 such that $\mathrm{conv}(X_1) \cap \mathrm{conv}(X_2) \neq \emptyset$. (4 points)

Due date: Thursday, May 2, 2019, before the lecture.