Summer term 2019 | Research Institute for Discrete Mathematics
Dr. U. Brenner | University of Bonn

Linear and Integer Optimization
Assignment Sheet 3

1. Let P be a polyhedron. Show that the problem of finding the largest ball that can be contained in P can be written as a linear program. (5 points)

2. Let $A \in \mathbb{R}^{m \times n}$. Show that exactly one of the systems

$$Ax = 0, \quad x > 0$$

and

$$A^t y \geq 0, \quad A^t y \neq 0$$

has a feasible solution. The condition "$x > 0$" means that each entry of the vector x must be positive. (5 points)

3. For an undirected graph G with vertex weights $c : V(G) \to \mathbb{R}$ consider the following LP relaxation of the Vertex Cover Problem:

$$\begin{array}{lll} & \min \sum_{v \in V(G)} x_v c(v) & \\ s.t. & x_v + x_w \geq 1 & \text{for } \{v, w\} \in E(G) \\ & x_v \geq 0 & \text{for } v \in V(G) \end{array}$$

Dualize this LP. In the unweighted case (i.e. $c(v) = 1$ for all $v \in V(G)$), the dual LP can be seen as the LP relaxation of a well-known combinatorial optimization problem. Which one? (4 points)

4. For $A \in \mathbb{R}^{m \times n}$, $c \in \mathbb{R}^n$ and $b = (b_1, \ldots, b_m) \in \mathbb{R}^m$ let $x^* \in \mathbb{R}^n$ be an optimum solution of the LP $\max\{c^t x \mid Ax \leq b\}$. Moreover, let $\tilde{b} = (\tilde{b}_1, \ldots, \tilde{b}_m) \in \mathbb{R}^m$, and let $\tilde{x} \in \mathbb{R}^n$ be a vector with $A\tilde{x} \leq \tilde{b}$. Prove that $\tilde{x}$ is an optimum solution of the LP $\max\{c^t x \mid Ax \leq \tilde{b}\}$ if $a_i^t \tilde{x} < \tilde{b}_i$ implies $a_i^t x^* < b_i$ for any $i \in \{1, \ldots, m\}$ (where a_i^t is the i-th row of A). (4 points)

5. Let $P, Q \subseteq \mathbb{R}^n$ be two polyhedra. Is it true that $\text{conv}(P \cup Q)$ is necessarily a polyhedron? Prove the correctness of your answer. (2 points)

Due date: Thursday, April 25, 2019, before the lecture.