Summer term 2019
Dr. U. Brenner

Research Institute for Discrete Mathematics
University of Bonn

Linear and Integer Optimization
Assignment Sheet 2

1. A paper mill produces paper rolls of 3 m width. The customers order rolls with smaller widths and the mill has to cut the ordered rolls out of the 3 m wide rolls. For example, a 3 m wide roll may be cut into two 93 cm wide and a 108 cm wide roll, leaving an offcut of 6 cm. The current order consists of

   - 90 rolls of width 130 cm,
   - 610 rolls of width 108 cm,
   - 395 rolls of width 42 cm, and
   - 211 rolls of width 93 cm.

   Formulate an integer linear program that minimizes the number of produced 3 m rolls and allows a correct cutting of the ordered rolls. (5 points)

2. (a) Let (P) be a linear program of the form $\max\{c^t x \mid Ax \leq b\}$. Show that the dual of the dual of (P) is equivalent to (P).

   (b) Consider the following linear program $\min\{c^t x \mid Ax = b\}$. Show that it either does not have a solution, it is unbounded, or all feasible solutions are optimal. (2+2 points)

3. Let $A \in \mathbb{R}^{m\times(n+k)}$ and $b \in \mathbb{R}^m$. Show that

$$P = \{x \in \mathbb{R}^n \mid \exists y \in \mathbb{R}^k : A\begin{pmatrix} x \\ y \end{pmatrix} \leq b\}$$

   is a polyhedron. (5 points)

4. Let $A \in \mathbb{R}^{m\times n}$, $c \in \mathbb{R}^n$ and $b, \tilde{b} \in \mathbb{R}^m$. Conside the following linear programs:

   (P1) $\max\{c^t x \mid Ax \leq b, x \geq 0\}$

   (P2) $\max\{\mathbb{1}_n^t x \mid Ax \leq b, x \geq 0\}$

   (P3) $\max\{c^t x \mid Ax \leq \tilde{b}, x \geq 0\}$

   Which of the following statements are necessarily true? Prove the correctness of your answers.

   (a) If (P1) is unbounded then (P2) is unbounded.

   (b) If (P2) is unbounded then (P1) is unbounded.

   (c) If (P1) is unbounded then (P3) is infeasible or unbounded. (2+2+2 points)

Due date: Thursday, April 18, 2019, before the lecture.