Research Institute for Discrete Mathematics
University of Bonn

Linear and Integer Optimization
Assignment Sheet 1

1. A company produces and sells two different products. Our goal is to determine the number of units of each product they should produce during one month, assuming that there is an unlimited demand for the products, but there are some constraints on production capacity and budget.

   There are 20000 hours of machine time in the month. Producing one unit takes 3 hours of machine time for the first product and 4 hours for the second product. Material and other costs for producing one unit of the first product amount to 3 MU (monetary units), while producing one unit of the second product costs 2 MU. The products are sold for 6 MU and 5 MU per unit, respectively. The available budget for production is 4000 MU initially. 25% of the income from selling the first product can be used immediately as additional budget for production, and so can 28% of the income from selling the second product.

   (a) Formulate a linear program to maximize the profit subject to the described constraints.

   (b) Solve the linear program graphically by drawing its set of feasible solutions and determining an optimal solution from the drawing.

   (c) Suppose the company could modernize their production line to get an additional 2000 machine hours for the cost of 400 MU (this would in particular reduce the available budget). Would this investment pay off? (2+2+2 points)

2. Let two finite disjoint sets A and B of vectors in $\mathbb{R}^2$ be given. We ask for a quadratic function $f(x) = a_2x^2 + a_1x + a_0$, such that all points in A are below the curve $\{(x, y) \mid x \in \mathbb{R}, y = f(x)\}$ and all point in B are above that curve. Describe a linear program whose solution allows you to decide directly if such a polynomial exists and, if it exists, to compute one. (4 points)

3. Specify necessary and sufficient conditions for the numbers $\alpha, \beta, \gamma \in \mathbb{R}$ such that the linear program $\max\{x + y \mid \alpha x + \beta y \leq \gamma, x \geq 0, y \geq 0\}$

   (a) has an optimum solution,

   (b) has a feasible solution,

   (c) is unbounded. (2+2+2 points)

4. Prove that $\operatorname{conv}(X)$ is the smallest convex set containing X. (4 points)

Due date: Thursday, April 11, 2019, before the lecture.