Summer term 2026
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Linear and Integer Optimization

Exercise Sheet 13

Exercise 13.1: A company needs to plan its production for then next n time-periods. We assume that each item of the product is indivisible (such as a boat, a computer, etc.). The production cost per item in period $t = 1, \ldots, n$ is c_t , while the demand at period t is d_t. The company may decide to store products from one period to the next; the unit cost of storing an item from period t to period $t + 1$ is h_t. Assume that there is no initial stock.
Write an integer programming problem to determine the minimum cost production plan. Show that the formulation is integral, by showing that the constraint matrix is totally unimodular. (5 bonus points)

Exercise 13.2: Prove Proposition 10.3 of the lecture notes, i.e. prove that if

$$\begin{array}{lrcl} \max & c^t x & & \\ \text{s.t.} & Ax + w & = & b \\ & x, w & \geq & 0 \end{array}$$

has a bounded and non-empty set of feasible solutions then

$$\mathcal{F}^o(D) := \{(y, z) \in \mathbb{R}^{m+n} \mid y^t A - z = c; y, z > 0\}$$

is non-empty. (5 bonus points)

The points on this sheet do not count for the 100 % you can get.

Submission deadline: Tuesday, July 21, 2026, 16:00, via eCampus (in groups of at most 3 students).