Summer term 2026 Research Institute for Discrete Mathematics
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Linear and Integer Optimization

Exercise Sheet 7

Exercise 7.1: Let $A := \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ s & -1 \end{pmatrix}$ and $b := \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

Use the IDEALIZED ELLIPSOID ALGORITHM with $R = 2$ to compute a feasible solution in $P = \{x \in \mathbb{R}^2 \mid Ax \leq b\}$ for $s = -1$ and for $s = -2$. (4 points)

Exercise 7.2: Let $P \subset \mathbb{R}^d$ be a finite non-empty set of points and let B be a ball containing P. Show: B is a ball of minimum radius containing P if and only if the center of B lies in $\mathrm{conv}(P \cap \partial B)$, where ∂B is the border of the ball. (5 Points)

Exercise 7.3: A semidefinite program is an optimization problem

$$\begin{aligned} \min\ & C \star X & \\ & A_i \star X \leq b_i & \forall i = 1, \dots, m \\ & X \succeq 0 & \\ & X \in \mathbb{R}^{n \times n} & \end{aligned}$$

where $C, A_1, \dots, A_m$ are matrices, $A \star X := \sum_{1 \leq i,j \leq n} a_{ij} x_{ij}$ and $X \succeq 0$ means that X is symmetric and positive semidefinite.

1. Show that the set $\{X \in \mathbb{R}^{n \times n} \mid X \succeq 0\}$ is a closed cone.
2. Construct a polynomial-time separation oracle for this set. (You may assume that you can compute basic arithmetic operations on real numbers, including square roots, exactly and in constant time.) (3+3 Points)

Exercise 7.4: Let G be a simple undirected graph. Consider the following linear program:

$$\begin{array}{lrcll} \min & \sum_{e=\{v,w\} \in E(G)} x_{vw} & & & \\ \text{s.t.} & \sum_{w \in S} x_{vw} & \geq & \lceil \frac{1}{4}|S|^2 + \frac{1}{2}|S| \rceil & \text{for } v \in V(G), S \subseteq V(G) \setminus \{v\} \\ & x_{uw} & \leq & x_{uv} + x_{vw} & \text{for } u, v, w \in V(G) \\ & x_{vw} & \geq & 0 & \text{for } v, w \in V(G) \\ & x_{vv} & = & 0 & \text{for } v \in V(G) \end{array}$$

(a) Show that this is a relaxation of the following problem: Find distances x_{vw} for the nodes of G such that $\sum_{e=\{v,w\} \in E(G)} x_{vw}$ is minimized under the condition that there is an ordering $\{v_1, \dots, v_{|V(G)|}\} = V(G)$ with $x_{v_i v_j} = |i - j|$ for $i, j \in \{1, \dots, |V(G)|\}$.

(b) Prove that there is a polynomial-time separation oracle for the polyhedron of the feasible solutions of the LP. (3+2 Points)

Submission deadline: Tuesday, June 9, 2026, 16:00, via eCampus (in groups of at most 3 students).